Simulation Equation Summary

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This document is written with Simulation, Flight Control, and Flight Data Analysis in mind.

# Equations of Motion

## Reference Frames and Coordinate Systems

I – Earth Centered Inertial (ECI)

Non-rotating frame, Origin fixed at Earth Center

E – Earth Centered Earth Fixed (ECEF)

ECEF frame rotates with respect to ECI frame, Origin fixed at Earth Center

G – Geographic/geocentric

Spherical earth model, coordinates are in lat, long, alt

D – Geodetic (ex. WGS84)

Oblate type earth model, WGS84 is an example in use by GPS, coordinates are in lat, long, alt

L – Local level

Local Level frame, origin at aircraft CG, x-North, y-East, z-Down

A – Airmass (Atmospheric = wind + gust + turbulence)

B – Body

S – Stability

W – Wind

## Conventions and Notation

T – Coordinate transformation matrix

( - Transformation matrix from coordinate system A to system B)

R – Rotation matrix

( - Rotation matrix of the 1-axis of frame B wrt frame A)

a – Acceleration

( – Acceleration of frame B wrt frame I, in coordinate system B)

v – Velocity

r – Position

α – Rotation acceleration

ω – Rotation rate

s – Orientation

Some of the common notation for flight dynamic models defined with this notation system.

Euler rotations from L to B:

Where:

Particular definitions common in flight dynamics:

Body Rotation Rates:

Body Accelerations:

Body Airspeed Velocities:

## Fundamental Definitions

### Derivatives (Euler Transformation)

Relating vectors that are differentiated in different coordinate systems is handled via the Euler Transformation.

The coordinate system used for the derivative can be denoted in two ways:

If the derivative is taken in a coordinate system that is not the reference system, then we use the Euler Transformation:

Examples:

### Coordinate Transformations

The “standard” Euler angle sequence for aircraft is the 3-2-1 sequence from the Local Level frame to the Body frame with intrinsic intermediate transformations. The sequence starts aligned to the Local Level frame rotate the 3-axis by angle , the new 2-axis by angle , and finally the 1-axis by angle . We can denote the transformation in the abbreviated notation, long form, or as a sequence of intrinsic rotations (note the order). Generally the abbreviated form is preferred, the long form is used were clarity is required.

Where:

Coordinate transformations can be easily and simply reversed by a simple transpose.

Also, note that

Which can lead to great confusion!

### Relative Velocity and Acceleration

General Relative Velocity/Acceleration of Frame B wrt Frame O from knowledge of Velocity/Acceleration of Frame A wrt Frame O. This is treated in the general case. The coordinate system needs to be consistent, but is not limited to any particular coordinate system, here the coordinate system is denoted with \*.

In the case that Frame B and Frame A are on a rigid body;

and

This reduces the equations to:

### Euler Equations (Conservation of Momentum)

### Differentiation of Transformation Matrix

Where the skew symmetric matrix:

Note:

Examples:

## Common Transformations

### Earth Center Earth Fixed (E) to/from Earth Centered Inertial (I)

(Depends on model. For Non-rotating earth models the two frames are the same)

### Earth Center Earth Fixed (E) to/from Geocentric (G)

Where: – Geocentric longitude (rad)

– Geocentric latitude (rad)

### Earth Center Earth Fixed (E) to/from Geodetic (D)

The conversion from/to Geodetic is non-linear transformation.

**Geodetic to ECEF**

For WGS84: a = 6378137.0, = 1/298.257223563

**ECEF to Geodetic**

(Iterative approximation)

### Earth Center Earth Fixed (E) to/from Local Level (L)

The Local Level frame is measured from a reference point, the X and Y coordinates of the L frame form a plane that is locally tangent to the geoid. Generally the reference point is taken on the surface of the geoid, but this is not necessary. The transformations are specific to the coordinate definition of the L frame, here North, East, and Down are the coordinates of the L frame.

### Geocentric (G) to/from Geodetic (D)

(Need Equations)

## Note on Quaternions

# Simulation

## Simulation Initialization

Initial velocity (m/s):

Initial position (deg, deg, m):

Initial rotation rate (rad/s):

Initial Euler angles (rad):

Initial Airspeed (m/s):

## Inputs

Mass (kg):

Mass Change Rate (kg/s) :

Inertia Tensor (N-m^2) :

Inertia Tensor Change Rate (N-m^2/s) :

Acceleration due to Gravity (m/s^2) :

External Forces (N) :

External Moments (N-m) :

Gust Velocity Disturbances (m/s) :

Gust Rotation Disturbances (m/s) :

## Conservation of Angular Acceleration

Where: are the External Moments acting on the Body

(Non-Rotating Earth) (Flat Earth – Level Local Plane does not rotate)

(Non-Rotating Earth)

(Flat Earth – Level Local Plane does not rotate)

## Integration of Angular Acceleration

Initialize:

(Non-Rotating Earth)

(Flat Earth – Level Local Plane does not rotate)

Gust Disturbance:

(By Definition)

(Non-Rotating Earth)

(Flat Earth – Level Local Plane does not rotate)

## Integration of Angular Rate (via Quaternion)

(Normalized)

Initialize:

## Conservation of Translational Momentum

Where: are the External Forces acting on the Body

(Definition)

(Flat Earth – Level Local Plane does not rotate)

## Integration of Translational Acceleration

Initialize:

(Definition)

Gust Disturbance:

(Definition)

(Flat Earth – Level Local Plane does not rotate)

## Integration of Translational Velocity

Initialize: (Tracking relative motion with the Local Level assumption)

(Level Local Plane defined at Initial Geodetic)

## Auxiliary Equations

Auxiliary Equations augment the EOM outputs with additional signals required elsewhere in the simulation.

### Navigation

**Geoid**

A Geoid model is required to provide the altitude above Mean Seal Level (MSL). A model such as the EGM-96 is applied. Geodetic coordinates are required input, MSL altitude is output.

**Terrain**

A Terrain model is required to provide altitude Above Ground Level (AGL). The ground elevation in Geodetic coordinates is used from the Environment Model. Currently a simple constant is used currently. A higher order Geoid model could be employed.

**Misc**

Flight path signals are generated to aide in debugging and performance checking. Ground speed, flight path angle (), and flight path course () are computed.

### Flow Conditions

The inflow conditions are computed. These are generally required for the Aerodynamic Models. Note that the **steady** component of the wind is applied prior to computation of the flow conditions.

**Apply Wind**

Where: (velocity of the wind in NED coordinates)

Where: (acceleration of the wind in NED coordinates)

**Inflow Signals**

True Airspeed:

Angle of Attack:

Angle of Flank:

Angle of Sideslip:

Mach Number:

Dynamic Pressure:

**Inflow Rate of Change**

True Airspeed Rate:

Angle of Attack Rate:

Angle of Flank Rate:

Angle of Sideslip Rate:

# Measurands and Sensors

For the purposes of Simulation and Flight Data Analysis, the sensor measurements need to estimate parameters at a common reference location on the aircraft. Generally this reference location is the nominal CG location, which coincides with the origin of the Body frame. In addition the sensors are not ideal and therefor error models are attributed to them. The perspective of the sensors is reversed between the Simulation sense and the Flight Data Analysis Sense. For example, in the Sim we go from the Body acceleration and compute the acceleration at the sensor then apply a sensor error/response model. For Flight Data Analysis we attempt to estimate the Body acceleration from the imperfect acceleration and transform those measurements to the Body frame.

The descriptions here-in approach sensors and measurands from the perspective of the Simulation. Caveats for Flight Data Analysis are discussed where appropriate.

## Sensor Transformation and Error Models

Sensor error models are described generically. Sensor transformations to the Body or Inertial Frames are handled separately.

### Accelerometer

Accelerometers measure acceleration, and they are sensitive to the gravitational acceleration at rest (for this reason they often treated as measuring “specific force”). Our convention is that if the accelerometer frame is aligned to the local level frame, the z component of acceleration is -1 g (-9.81 m/s^2), and x and y components are 0.0. There are several ways to account for this in the measurement prior to integration in the equations of motion. The complication is that the orientation of the accelerometer (or body frame) generally needs to be know wrt the Local Level (or inertial) frame in order to remove the gravitation effect.

For simulation we know and need to compute the acceleration at the sensor location. Then the effect of gravity is applied, and finally the error model is applied.

Fir flight data analysis the order is reversed. First the error model is applied to estimate the “true” measurand, the effect of gravity is removed to yield a true acceleration, then finally the relative acceleration is computed at the Body frame.

**Relative Measurement**

**Effect of gravity**

Remove the effect of gravity. This is done here by modeling a difference in acceleration between the Inertial frame and the local level frame.

Where:

Where:

**Sensor Error**

Where:

=

– Off-axis sensitivity

– Scale Factor

– Bias

– Noise

Definition of T, the coordinate rotation from the Accel system to the Body System, and MS are not linearly independent. T is intended to only capture the installation mis-alignment between the Body frame and the Accelerometer Frame. As such it is parameterized by three rotation angles. MS is intended to capture Scale Factor calibration errors in the sensor. For most Accelerometers (and Gyros) the values for MS should be negligible as the manufacturer compensates the sensor output for these factors. The exception is that M and S can have a temperature sensitivity which may not be well compensated, hence temperature compensation can be introduced as a linear function affecting the elements of M and S.

Bias (B) and noise (V) are unavoidable. Noise could be modeled as simple white noise or as a more complicated noise function.

For flight data analysis and system identification in particular the redundant nature of T and MS causes a collinearity if both parameters are attempted to be estimated. Because installation mis-alignment of the IMU package is anticipated to be dominate it is recommended, as a first step, to attempt to estimate T and B while setting MS to zero.

### Gyro

**Relative Measurement**

Rotation Rate is a “free” vector in a rigid body. ( )

**Sensor Error**

– off-axis sensitivity

– Scale Factor

– Gyro sensitivity to linear acceleration

– Bias

– Random Noise

### Magnetometer

Magnetometers can indicate an orientation wrt the world magnetic frame.

**Relative Measurement**

**Sensor Error**

### Air data

The airdata sensors indirectly estimate the relative velocity of the vehicle frame (B) wrt the surrounding Air Mass frame (A). For the pitot-static system this is done with measurement of pressures.

**Relative Measurement**

A typical pitot-static system is insensitive to small changes in angle of the velocity vector to about 10 degrees. Therefor nose booms are insensitive to body rotations. Wingtip booms are sensitive to yaw rate, though the impact should be small in most applications. Angle vanes measure small changes in in-flow condition and multi-hole probes are very sensitive to even small angle changes.

**Measurand Computation**

True Airspeed:

Dynamic Pressure:

Angle of attack:

Angle of Flank:

Angle of Sideslip:

Static Pressure:

Temperature:

**Sensor Error**

The pitot-static system measures pressures, these pressures are then converted to velocity and altitude estimates. The pressure system error model for both transducers is a function of the local dynamic pressure. The scale factors, k, can be temperature dependent (should be compensated by manufacturer).

### GPS

Many GPS provide an estimate of time, position, and an independent estimate of velocity. Position measurements are wrt either the ECEF or WGS84 frame. Velocity measurements are typically either wrt ECEF or the Local Level frame. These estimates are at the antenna location, and are invariant to antenna orientation.

**Relative Measurement**

Where:

Where:

**Sensor Error**

# Subsystem Models

## Control Surface Model

### Servo

### Linkage

## Mass Properties Model

## Aerodynamic Model

### Coefficient Build-up

Where:

Where:

Where:

Where:

Where:

Where:

### Forces and Moments

### Transformation to Body Frame

## Propulsion Model

# Flight Control System

## Sensor Processing

The typical goal for the sensor processing stage is to estimate the equation of motion states. Typically, the processes is to:

1. Apply filtering
2. Transform measures to Body frame
3. Estimate additional states (for example with a Kalman Filter)

# Flight Data Analysis

## Sensor Processing

This is similar to the process outlined for the Flight Control System, however more filtering and estimation options are available since the entire time history is available.

## Airdata Calibration and Wind Estimation

1. Pre-Process Flight Data
2. Transfer gyro data to the Body frame
   1. Apply sensor error model corrections
   2. Relative Rotation Rate to the Body frame
   3. Rotate Coordinate system from Gyro to Body
3. Integrate gyro data to get Euler Angles
4. Validate Euler angles against EKF derived angles
5. Transform Accelerometer data to the Body frame
   1. Apply sensor error model corrections
   2. Relative Acceleration of Body Frame
   3. Remove gravity
   4. Rotate coordinate system from accel to Body
6. Integrate accelerometer data to get Body Velocity